

C4 INTEGRATION

Answers - Worksheet B

$$1 \quad \mathbf{a} = \frac{1}{8}(x-2)^8 + c \quad \mathbf{b} = \frac{1}{2} \times \frac{1}{4}(2x+5)^4 + c \quad \mathbf{c} = \frac{1}{3} \times \frac{6}{5}(1+3x)^5 + c \quad \mathbf{d} = 4 \times \frac{1}{6} \left(\frac{1}{4}x-2\right)^6 + c$$

$$= \frac{1}{8}(2x+5)^4 + c \quad = \frac{2}{5}(1+3x)^5 + c \quad = \frac{2}{3} \left(\frac{1}{4}x-2\right)^6 + c$$

$$\mathbf{e} = -\frac{1}{5} \times \frac{1}{5}(8-5x)^5 + c \quad \mathbf{f} = \int (x+7)^{-2} dx \quad \mathbf{g} = \int 8(4x-3)^{-5} dx \quad \mathbf{h} = \int \frac{1}{2}(5-3x)^{-3} dx$$

$$= -\frac{1}{25}(8-5x)^5 + c \quad = -(x+7)^{-1} + c \quad = \frac{1}{4} \times \frac{8}{-4}(4x-3)^{-4} + c \quad = -\frac{1}{3} \times \frac{1}{-4}(5-3x)^{-2} + c$$

$$= \frac{-1}{2(4x-3)^4} + c \quad = \frac{1}{12(5-3x)^2} + c$$

$$2 \quad \mathbf{a} = \frac{2}{5}(3+t)^{\frac{5}{2}} + c \quad \mathbf{b} = \int (4x-1)^{\frac{1}{2}} dx \quad \mathbf{c} = \frac{1}{2} \ln |2y+1| + c$$

$$= \frac{1}{4} \times \frac{2}{\frac{3}{2}}(4x-1)^{\frac{3}{2}} + c$$

$$= \frac{1}{6}(4x-1)^{\frac{3}{2}} + c$$

$$\mathbf{d} = \frac{1}{2}e^{2x-3} + c \quad \mathbf{e} = 3 \times \frac{1}{-\frac{7}{2}} \ln |2-7r| + c \quad \mathbf{f} = \int (5t-2)^{\frac{1}{3}} dt$$

$$= -\frac{3}{7} \ln |2-7r| + c \quad = \frac{1}{5} \times \frac{3}{\frac{4}{3}}(5t-2)^{\frac{4}{3}} + c$$

$$= \frac{3}{20}(5t-2)^{\frac{4}{3}} + c$$

$$\mathbf{g} = \int (6-y)^{-\frac{1}{2}} dy \quad \mathbf{h} = -\frac{5}{3}e^{7-3t} + c \quad \mathbf{i} = 4 \times \frac{1}{3} \ln |3u+1| + c$$

$$= -2(6-y)^{\frac{1}{2}} + c \quad = \frac{4}{3} \ln |3u+1| + c$$

$$3 \quad \mathbf{a} \quad f(x) = \int 8(2x-3)^3 dx$$

$$= \frac{1}{2} \times 2(2x-3)^4 + c$$

$$= (2x-3)^4 + c$$

$$(2, 6) \Rightarrow 6 = 1 + c$$

$$\therefore c = 5$$

$$f(x) = (2x-3)^4 + 5$$

$$\mathbf{b} \quad f(x) = \int 6e^{2x+4} dx$$

$$= 3e^{2x+4} + c$$

$$(-2, 1) \Rightarrow 1 = 3 + c$$

$$\therefore c = -2$$

$$f(x) = 3e^{2x+4} - 2$$

$$\mathbf{c} \quad f(x) = \int 2 - \frac{8}{4x-1} dx$$

$$= 2x - 8 \times \frac{1}{4} \ln |4x-1| + c$$

$$= 2x - 2 \ln |4x-1| + c$$

$$\left(\frac{1}{2}, 4\right) \Rightarrow 4 = 1 + c$$

$$\therefore c = 3$$

$$f(x) = 2x - 2 \ln |4x-1| + 3$$

$$\mathbf{d} \quad f(x) = \int 8x - 3(3x-2)^{-2} dx$$

$$= 4x^2 + \frac{1}{3} \times 3(3x-2)^{-1} + c$$

$$= 4x^2 + (3x-2)^{-1} + c$$

$$(-1, 3) \Rightarrow 3 = 4 - \frac{1}{5} + c$$

$$\therefore c = -\frac{4}{5}$$

$$f(x) = 4x^2 + \frac{1}{3x-2} - \frac{4}{5}$$

$$\begin{aligned}
 4 \quad \mathbf{a} &= \left[\frac{1}{3} \times \frac{1}{3} (3x+1)^3 \right]_0^1 \\
 &= \frac{1}{9} [(3x+1)^3]_0^1 \\
 &= \frac{1}{9} (64-1) \\
 &= 7
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} &= \left[\frac{1}{2} \times \frac{1}{4} (2x-1)^4 \right]_1^2 \\
 &= \frac{1}{8} [(2x-1)^4]_1^2 \\
 &= \frac{1}{8} (81-1) \\
 &= 10
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} &= \int_2^4 (5-x)^{-2} dx \\
 &= [(5-x)^{-1}]_2^4 \\
 &= 1 - \frac{1}{3} \\
 &= \frac{2}{3}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d} &= \left[\frac{1}{2} e^{2x+2} \right]_{-1}^1 \\
 &= \frac{1}{2} (e^4 - 1)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{e} &= \int_2^6 (3x-2)^{\frac{1}{2}} dx \\
 &= \left[\frac{1}{3} \times \frac{2}{3} (3x-2)^{\frac{3}{2}} \right]_2^6 \\
 &= \frac{2}{9} [(3x-2)^{\frac{3}{2}}]_2^6 \\
 &= \frac{2}{9} (64-8) \\
 &= 12\frac{4}{9}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{f} &= [4 \times \frac{1}{6} \ln |6x-3|]_1^2 \\
 &= \frac{2}{3} [\ln |6x-3|]_1^2 \\
 &= \frac{2}{3} (\ln 9 - \ln 3) \\
 &= \frac{2}{3} \ln 3
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{g} &= \int_0^1 (7x+1)^{-\frac{1}{3}} dx \\
 &= \left[\frac{1}{7} \times \frac{3}{2} (7x+1)^{\frac{2}{3}} \right]_0^1 \\
 &= \frac{3}{14} (4-1) \\
 &= \frac{9}{14}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{h} &= \left[\frac{1}{5} \ln |5x+3| \right]_{-7}^{-1} \\
 &= \frac{1}{5} (\ln 2 - \ln 32) \\
 &= \frac{1}{5} (\ln 2 - 5 \ln 2) \\
 &= -\frac{4}{5} \ln 2
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{i} &= \frac{1}{8} \int_4^7 (x-4)^3 dx \\
 &= \frac{1}{8} \left[\frac{1}{4} (x-4)^4 \right]_4^7 \\
 &= \frac{1}{32} (81-0) \\
 &= 2\frac{17}{32}
 \end{aligned}$$

$$\begin{aligned}
 5 \quad \mathbf{a} &= \int_3^4 e^{3-x} dx \\
 &= [-e^{3-x}]_3^4 \\
 &= -e^{-1} - (-1) \\
 &= 1 - \frac{1}{e}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b} &= \int_2^3 (3x-5)^3 dx \\
 &= \left[\frac{1}{3} \times \frac{1}{4} (3x-5)^4 \right]_2^3 \\
 &= \frac{1}{12} (256-1) \\
 &= 21\frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c} &= \int_1^4 \frac{3}{4x+2} dx \\
 &= \left[3 \times \frac{1}{4} \ln |4x+2| \right]_1^4 \\
 &= \frac{3}{4} (\ln 18 - \ln 6) \\
 &= \frac{3}{4} \ln 3
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d} &= \int_{-2}^0 (1-2x)^{-2} dx \\
 &= \left[-\frac{1}{2} \times -(1-2x)^{-1} \right]_{-2}^0 \\
 &= \frac{1}{2} \left(1 - \frac{1}{5} \right) \\
 &= \frac{2}{5}
 \end{aligned}$$

$$\begin{aligned}
 6 &= \int_0^1 12(2x+1)^{-3} dx \\
 &= \left[\frac{1}{2} \times (-6)(2x+1)^{-2} \right]_0^1 \\
 &= \left[\frac{-3}{(2x+1)^2} \right]_0^1 \\
 &= -\frac{1}{3} - (-3) \\
 &= \frac{8}{3}
 \end{aligned}$$